

Important info: I'm not even worrying about models with the same BS, because it's a pain in the ass. If this is useful to you, *then* I'll figure out that case.

Consider a face-to-face roll between two combatants with BS values  $\beta_1$  and  $\beta_2$  and burst values  $b_1$  and  $b_2$ . Let's forget about criticals for a sec, they make everything a pain in the ass. We want to know how many combinations of  $b_1 + b_2$  dice result in  $k$  successes for player 1. First of all, it's ambiguous *which* of player 1's dice succeed. Let's say it's the first  $k$  and multiply our answer by  $\binom{b_1}{k}$ . Now, one case is easy right off the bat. Let's say all of player 2's shots missed. The number of combinations of dice that result in that particular scenario is:

$$\binom{b_1}{k} \beta_1^k (20 - \beta_1)^{b_1 - k} (20 - \beta_2)^{b_2}$$

It's harder if some of player 2's shots hit, but were overridden by player 1's success. In that case, the possible values of a successful roll for player 1 are constrained - only rolls between player 2's maximum roll and player 1's BS will yield success. Let's pretend to ourselves we know 2 numbers:  $m$ , player 2's maximum roll, and  $n_m$ , the number of dice with which player 2 rolled that number. Again, there's ambiguity about which  $n_m$  dice, contributing  $\binom{n}{n_m}$  possibilities. In this case, how many values of player 1's dice are successes. Well, if  $\beta_1 > \beta_2$ , then player 1 wins ties, so it's  $\beta_1 - m + 1$ . Otherwise, it's  $\beta_1 - m$ . Let's define  $T(\beta_1, \beta_2)$  to be 1 when  $\beta_1 > \beta_2$ , 0 otherwise. Then, defining  $S = \beta_1 - m + T$  and  $F = 20 - \beta_2 + m - 1$  (the rest of player 2's dice can either be misses or less than  $m$ )

$$\binom{b_1}{k} \binom{b_2}{n_m} S^k (20 - S)^{b_1 - k} F^{b_2 - n_m}$$

From here, we have to recognize that individual values of  $m$  and  $n_{max}$  are mutually exclusive. Because of that, we can sum over them without double-counting anything. They're also exhaustive - every combination of dice has some value of  $m$  and some value of  $n_{max}$ . So, we're there!

$$\beta_1 \beta_2 N_k^{b_1 b_2} = \binom{b_1}{k} \left( \beta_1^k (20 - \beta_1)^{b_1 - k} (20 - \beta_2)^{b_2} + \sum_{m=1}^{\min(\beta_1 - 1, \beta_2)} \sum_{n_m=1}^{b_2} \binom{m}{n_m} S^k (20 - S)^{b_1 - k} F^{b_2 - n_m} \right)$$

Now, we have to think about crits. Gross. Well, we already basically did one case, which is when there are zero crits, but  $k$  non-critical successes. Only small adjustment we need to make is that, now, a roll exactly equal to player 1's BS is neither a success nor a failure, and  $m$  can no longer range up to player 2's BS. So, now, let's define  $S' = S - 1$ . In this case, we have:

$$\beta_1 \beta_2 N_k'^{b_1 b_2} = \binom{b_1}{k} \left( (\beta_1 - 1)^k (20 - \beta_1 - 1)^{b_1 - k} (20 - \beta_2)^{b_2} + \sum_{m=1}^{\min(\beta_1 - 1, \beta_2 - 1)} \sum_{n_m=1}^{b_2} \binom{m}{n_m} S'^k (20 - S')^{b_1 - k} F^{b_2 - n_m} \right)$$

This will be useful in solving the other cases. The next-easiest case is  $c$  crits and  $k$  normal successes. Multiply by  $\binom{b_1}{c}$  to account for the ambiguity about which dice critted, and your left with a solved problem: player 1 now has burst  $b_1 - c$ , and we want to see how many combinations of  $b_1 - c + b_2$  dice result in  $k$  successes. But that's just  $\beta_1 \beta_2 N_k'^{b_1 - c, b_2}$ !

So, for the case  $k > 0$ ,

$$\beta_1 \beta_2 \Gamma_{ck}^{b_1 b_2} = \binom{b_1}{c} \beta_1 \beta_2 N_k'^{b_1 - c, b_2}$$

(I'm using  $\Gamma$  because that's what they use for number of microstates at a given energy in physics, and I can't think of anything better). Alright, now we're down to the cases where  $k = 0$ . In these

cases, player 1's remaining rolls were not successes. This could have happened for two reasons: either player 2 got some successes that were nullified by the crit(s), or both players whiffed completely. Let's define  $\beta_1\beta_2 M^{b_1b_2} = (20 - \beta_1)^{b_1}(20 - \beta_2)^{b_2}$ , the number of combinations of dice in which both players whiff. Now, first, let's do the case where  $\beta_1 < \beta_2$ . Why this one? Well, if player 1's BS is lower and s/he critted, you know that player 2 *didn't* crit. Keeping that in mind:

$$\beta_1\beta_2\Gamma_{ck}^{b_1b_2} = \binom{n}{c} \left[ \sum_{k'=1}^{b_2} \beta_2\beta_1 N_{k'}^{b_2, b_1-c} + \beta_2\beta_1 M^{b_2, b_1-c} \right]$$

Note that player 1 and 2's subscripts are switched in the  $N'$  there! So now we've almost got all the cases - we've got both players whiffing, we've got got  $c$  crits and  $k > 0$  successes for both players, and we've got  $c$  crits and  $k = 0$  successes for the player with the lower BS. Let's use these to calculate the last case:  $c$  crits and  $k = 0$  successes for the player with the higher BS. In this case, the other player *might* have critted. So, we have to add those possibilities in, too. We're left with:

$$\beta_1\beta_2\Gamma_{ck}^{b_1b_2} = \binom{n}{c} \left[ \sum_{k'=1}^{b_2} \beta_2\beta_1\Gamma_{0k'}^{b_2, b_1-c} + \sum_{c_2=0}^{b_2} \sum_{k'=0}^{b_2-c_2} \beta_2\beta_1\Gamma_{c_2k'}^{b_2-c_2, b_1-c} + \beta_2\beta_1 M^{b_2, b_1-c} \right]$$

This is completely in terms of stuff we already know, so we're done! I suspect this is the closest to closed form one can come for this. I also expect that the case where there's a draw shakes out like the easier of the last two cases, since crits knock each other out. Although ... what happens when the highest dice draw? I'm not actually sure. Anyways, hope someone enjoys reading this.